18.02A Recitation — Wednesday, Oct. 24, 2018

Quick Review

- Project a vector $\mathbf{v}$ onto another vector $\mathbf{w}$

$$\left(\mathbf{v} \cdot \frac{\mathbf{w}}{||\mathbf{w}||}\right) \frac{\mathbf{w}}{||\mathbf{w}||}$$

- Determinant of a 3-by-3 matrix
- Cross product $\mathbf{v} \times \mathbf{w}$ (works in 3D, the product is another 3D-vector)
    1. Geometrically: $||\mathbf{v} \times \mathbf{w}||$ is the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$; $\mathbf{v} \times \mathbf{w}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$ and the direction is determined by *right-hand rule.*
    2. Algebraically,

$$\langle x_1, y_1, z_1 \rangle \times \langle x_2, y_2, z_2 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Practice problems:

1. Project the vector $\langle 1, 2, 3 \rangle$ onto $\langle 1, 0, 1 \rangle$.

2. Compute the following determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}.$$

3. Compute the cross product

$$\langle 2, -1, 3 \rangle \times \langle -4, 3, -5 \rangle.$$

4. (Optional) Let $\mathbf{v} = \langle 2, -1, 3 \rangle$ and $\mathbf{w} = \langle -4, 3, -5 \rangle$. The two lines generated by $\mathbf{v}$ and $\mathbf{w}$ will determine a plane P.

   Suppose $\mathbf{A} = \langle 1, 2, 3 \rangle$. Find two vectors $\mathbf{B}$ and $\mathbf{C}$ such that $\mathbf{A} = \mathbf{B} + \mathbf{C}$ with $\mathbf{B}$ parallel to the plane P and $\mathbf{C}$ perpendicular to the plane P.