18.02A Recitation — Wednesday, Nov. 7, 2018

Quick Review

- Given a function $z = f(x,y)$. The *tangent plane* to its graph at (x_0, y_0, z_0) where $z_0 = f(x_0, y_0)$ is given by

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

- Linear approximation. Let $\Delta x = x - x_0$, $\Delta y = y - y_0$, $\Delta z = z - z_0$. Then

$$\Delta z \approx f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y.$$

Practice problems:

1. Consider $f(x,y) = \frac{y^2}{x}$. Find the tangent plane to the its graph at the point $(1,2)$.

2. To determine the volume of a cylinder of radius around 2 and height around 3, about how accurately should the radius and height be measured for the error in the calculated volume not to exceed 0.1?

3. Consider $z = x^2(y+1)$.

   (a) Around the point $(1, 0)$, is z more sensitive to changes in x or in y?

   (b) Around the point $(1, 0)$, what should the ratio of Δy to Δx be in order that small changes with this ratio produce no change in z, i.e., no first-order change—of course z will change a little, but like $(\Delta x)^2$, not like Δx.