18.02A Recitation — Monday, Nov. 26, 2018

Quick Review

- Second derivative test. Suppose (x_0, y_0) is a critical point for $z = f(x, y)$. Compute

$$A = f_{xx}(x_0, y_0), \qquad \Delta = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0)^2.$$

  - $A > 0, \Delta > 0$: (x_0, y_0) is a local min.
  - $A < 0, \Delta > 0$: (x_0, y_0) is a local max.
  - $\Delta < 0$: (x_0, y_0) is a saddle point.
  - $\Delta = 0$: inconclusive.

- Lagrange multiplier (for constrained optimization problem).

  Maximize/minimize $z = f(x, y)$ subject to the constraint $g(x, y) = c$, where c is a given constant.

  Introduce the additional variable λ (Lagrange multiplier) and solve

$$\begin{cases} g(x,y) &= 0 \\ f_x(x,y) &= \lambda g_x(x,y) \\ f_y(x,y) &= \lambda g_y(x,y). \end{cases}$$

  Compare the value of f at solutions to the above equations.

  This also extends to functions of three variables and you will have one more equation involving the partial derivative of the third variable.

  For two constraints, introduce an additional variable μ. See lecture note 12.

- Optimization problem over a bounded region R.
  - Step 1: find interior critical points.
  - Step 2: use Lagrange multiplier to study the function over the boundary of R.
  - Step 3: compare the values of f over the points you found in Steps 1 and 2.

Practice problems:

1. Let $f(x, y) = x^4 + y^4 - 4xy + 1$. Find the critical point(s) for f and determine if they are local min/max or saddle point(s).

2. What is the maximum and minimum value of $5x - 3y$ on the constraint set $x^2 + y^2 = 136$?

3. What is the min and max value of $81x^2 + y^2$ on the set $4x^2 + y^2 \leq 9$?

4. Find min and max value of $4y - 2z$ on the constraint set $2x - y - z = 2$, $x^2 + y^2 = 1$.