18.02A Recitation — Wednesday, Nov. 21, 2018

Quick Review

- Second derivative test. Suppose (x_0, y_0) is a critical point for $z = f(x, y)$. Compute

$$A = f_{xx}(x_0, y_0), \qquad \Delta = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0)^2.$$

  - $A > 0, \Delta > 0$: (x_0, y_0) is a local min.
  - $A < 0, \Delta > 0$: (x_0, y_0) is a local max.
  - $\Delta < 0$: (x_0, y_0) is a saddle point.
  - $\Delta = 0$: inconclusive.

- Lagrange multiplier (for constrained optimization problem).

  Maximize/minimize $z = f(x, y)$ subject to the constraint $g(x, y) = c$, where c is a given constant.

  Introduce the additional variable λ (Lagrange multiplier) and solve

$$\begin{cases} g(x,y) &= 0 \\ f_x(x,y) &= \lambda g_x(x,y) \\ f_y(x,y) &= \lambda g_y(x,y). \end{cases}$$

  Compare the value of f at solutions to the above equations.

  This also extends to functions of three variables and you will have one more equation involving the partial derivative of the third variable.

- Optimization problem over a bounded region R.
  - Step 1: find interior critical points.
  - Step 2: use Lagrange multiplier to study the function over the boundary of R.
  - Step 3: compare the values of f over the points you found in Steps 1 and 2.

Practice problems:

1. Let $f(x, y) = x^4 + y^4 - 4xy + 1$. Find the critical point(s) for f and determine if they are local min/max or saddle point(s).

2. What is the maximum and minimum value of $5x - 3y$ on the constraint set $x^2 + y^2 = 136$?

3. What is the min and max value of $81x^2 + y^2$ on the set $4x^2 + y^2 \leq 9$?