18.02A Recitation — Wednesday, Dec. 5, 2018

Quick Review

- To use polar coordinates to compute a double integral

$$\iint_R f(x,y)dxdy$$

  1. Rewrite the integrand into $f(r\cos\theta, r\sin\theta)$.
  2. $dxdy$ becomes $rdrd\theta$.
  3. Rewrite the region R using polar coordinates. Usually, use radial stripes:
     (a) Figure out the largest and the smallest θ for all points in R—your answer should be $a \le \theta \le b$.
     (b) Now fix an arbitrary θ, draw the radial slice intersecting the region R, and ask yourself: on this particular slice, what is the smallest r and what is the largest r?—your answer should be $g(\theta) \le r \le h(\theta)$.
  4. Now, put everything together:

$$\int_a^b \left(\int_{g(\theta)}^{h(\theta)} f(r\cos\theta, r\sin\theta) r dr \right) d\theta.$$

- Polar coordinates is useful especially when either your region is "round" or when your integrand has something like $x^2 + y^2$.

Practice problems:

1. Consider the circle C of radius 1 centered at $(0,1)$:

$$x^2 + (y-1)^2 = 1.$$

   (a) Use polar coordinates r and θ to represent C.

(b) Denote by R the region enclosed by the circle—so, R is the disk of radius 1 centered at $(0, 1)$. The area of R is given by

$$\iint_R 1dA.$$

Use polar coordinates to compute this double integral. (Your answer should be π, which is precisely the area of a unit disk.)

2. Consider two circles of radius 1, one centered at $(0, 0)$ and the other centered at $(1, 0)$. Let R be the region inside both circles. The area of R can be given by

$$\iint_R 1dA.$$

Use polar coordinates to compute this double integral.