18.02A Recitation — Monday, Oct. 29, 2018

Quick Review

- There are many parametric equations that describe the same curve. For example, both $\mathbf{r}_1(t) = t\,\mathbf{i} + 5t\,\mathbf{j}$ for $-\infty < t < \infty$ and $\mathbf{r}_2(s) = \arctan s\,\mathbf{i} + 5\arctan s\,\mathbf{j}$ for $-\pi/2 < s < \pi/2$ traces the same line $y = 5x$.
- Common parametric curves:
  1. Lines and ellipses. (You should be able to parameterize them and also recognize them if you are given the parametric equation).
  2. Helix
  $$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle.$$
  3. Cycloid
  $$\mathbf{r}(t) = \langle tv - a\sin(v/at), a - a\cos(v/at) \rangle,$$
  where v is the speed of the wheel in the x direction and a is the radius of the wheel.
- Given the position vector $\mathbf{r}(t)$,
  - velocity vector: $\mathbf{r}'(t)$
  - speed: $||\mathbf{r}'(t)||$
  - acceleration vector: $\mathbf{r}''(t)$

Practice problems:

1. Sketch the curve $x(t) = \sin t$, $y(t) = -3 + 2\cos t$. In which direction is the object moving?

2. A rod of length a is placed on the plane with one end fixed at the origin. The rod is rotating counterclock-wise at an angular speed of ω_1 radians/second. A smaller rod of length b (with $b < a$) has its one end fixed at the other end of the first rod. The smaller rod is rotating clock-wise at an angular speed of ω_2 radians/second. Initially, the non-fixed end of the first rod is at $(a, 0)$ whereas the non-fixed end of the second rod is at $(a - b, 0)$. Find the position vector of the non-fixed end of the second rod.

3. An object P is moving in space with position vector $\mathbf{r}(t) = \langle 3\cos t, 5\sin t, 4\cos t\rangle$.

   (a) Show that P moves on the surface of a sphere.

   (b) Show that its speed is constant.

   (c) Show that the direction of the acceleration vector is towards the origin; that is, show $\mathbf{a} = -\mathbf{r}$.

   (d) Show that P is also moving on a plane that contains the origin.

   (The trajectory of P is hence the intersection of a centered sphere and a plane through the origin—in another word, a great circle.)