18.01A Recitation — Wednesday, Sept. 12, 2018

Quick Review:

- Definite integral $\int_a^b f(x)dx$
  - Geometric meaning: "signed" area under the curve $y = f(x)$ between $x = a$ and $x = b$.
  - Algebraically: limit of the Riemann sum as the partition gets finer.
    1. partition $[a, b]$ into n equal intervals with length $\Delta x = \frac{b-a}{n}$
    2. pick a point inside each interval, $c_i \in [x_i, x_{i+1}]$
    3. pretend that inside each interval, the function is constantly $f(c_i)$, which leads to
    $$\sum_{i=0}^{n-1} f(c_{i+1})\Delta x$$
    4. take limit as $n \to \infty$
  - types of Riemann sum: left sum, right sum, upper sum, lower sum, ...
- Properties of definite integral:
  1. $\int_a^b c_1 f(x) + c_2 g(x)dx = c_1 \int_a^b f(x)dx + c_2 \int_a^b g(x)dx$
  2. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
  3. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- Fundamental Theorem of Calculus: if $F'(x) = f(x)$, then
$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

Practice problems:

1. Find $\int_0^1 e^x dx$ by using Riemann sum.

2. Find the limit by relating it to Riemann sum:

$$\lim_{n\to\infty} \frac{\cos(1/n) + \cos(2/n) + \cdots + \cos((n-1)/n) + \cos(1)}{n}$$

3. In class, we saw that for continuous functions, the limit of the Riemann sum does not depend on the choice of c_i inside each interval. For non-continuous functions, the story can **sometimes** be different. Consider the function $h(x)$ defined on $[0, 1]$:

$$h(x) = \begin{cases} 1 & \text{if } x \text{ is rational;} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that one cannot define $\int_0^1 h(x)dx$ by showing that the limit of the upper sum and the limit of the lower sum are different.